

Core 4 - January 2009

① a) i) $f(-1) = 4(-1)^3 - 7(-1) - 3 = 0$

ii) $2x+1 \rightarrow$ sub in $x = -1/2$

$f(-1/2) = 4(-1/2)^3 - 7(-1/2) - 3$ ~~is not~~ \therefore FACTOR

$\rightarrow 4(-1/8) + 7/2 - 3 = 0$

iii) Need 3rd factor

$(x+1)(2x+1)(\quad) = 4x^3 + 0x^2 - 7x - 3$

$(2x^2 + 3x + 1)(\quad) = 4x^3 + 0x^2 - 7x - 3$

$\begin{matrix} \nearrow & \nearrow \\ 2x & -3 \end{matrix}$

$\rightarrow \frac{(x+1)(2x+1)(2x-3)}{(2x+1)(2x+1)} = 2x-3$

b) $2x+1 \rightarrow$ sub in $x = -1/2$

using remainder theorem:

$g(-1/2) = 4(-1/2)^3 - 7(-1/2) + d = 2$

$-1/2 + 7/2 + d = 2$

$\rightarrow d = -1$

② a) $\sin(x) - 3\cos(x) = R \sin(x - \alpha)$

$= R \sin(x) \cos(\alpha) - R \cos(x) \sin(\alpha)$

$R = \sqrt{1^2 + 3^2} = \sqrt{10}$

$1 = R \cos(\alpha) \rightarrow \tan(\alpha) = 3/1 = 3$

$3 = R \sin(\alpha) \rightarrow \alpha = 1.249 \text{ (rad)}$

$\rightarrow \sqrt{10} \sin(x - 1.249)$

b) i) Minimum = $-\sqrt{10}$ as curve stretched SF $\sqrt{10}$ in y

ii) $y = \sin(x)$ has minimum value at $x = 3\pi/2$

Function has been translated 1.249 to right

\therefore new minimum = $3\pi/2 + 1.249 = 5.961\dots$

$= 5.96 \text{ (2dp)}$

3) a) i) $\frac{2x+7}{x+2} = A + \frac{B}{x+2}$

$\rightarrow 2x+7 = A(x+2) + B$

$x = -2$

$B = B$

$x = 0$

$7 = 2A + B \rightarrow A = 2$

$\rightarrow 2 + \frac{3}{x+2}$

ii) $\int \left(2 + \frac{3}{x+2} \right) dx = 2x + 3 \ln|x+2| + C$

b) i) $\frac{28 + 4x^2}{(1+3x)(5-x)^2} = \frac{P}{1+3x} + \frac{Q}{5-x} + \frac{R}{(5-x)^2}$

$28 + 4x^2 = P(5-x)^2 + Q(1+3x)(5-x) + R(1+3x)$

$x = 5$

$128 = R(16) \rightarrow R = 8$

$x = -1/3$

$256/9 = P(256/9) \rightarrow P = 1$

$x = 0$

$28 = (1)(5)^2 + Q(1)(5) + 8(1)$

$\rightarrow 28 = 25 + 5Q + 8$

$\rightarrow 5Q = -5 \rightarrow Q = -1$

$\rightarrow \frac{1}{1+3x} - \frac{1}{5-x} + \frac{8}{(5-x)^2}$

ii) $\int \frac{1}{1+3x} - \int \frac{1}{5-x} + 8 \int \frac{1}{(5-x)^2}$

$8 \int (5-x)^{-2}$

$\rightarrow \frac{1}{3} \ln|1+3x| - (-\ln|5-x|) + 8(5-x)^{-1} + C$

$\rightarrow \frac{1}{3} \ln|1+3x| + \ln|5-x| - \frac{8}{5-x} + C$

4) a) i) $(1-x)^{1/2} \approx 1 + (1/2)(-x) + \frac{(1/2)(-1/2)(-x)^2}{2!}$

$= 1 - 1/2x - 1/8x^2$

ii) $\sqrt{4-x} = (4-x)^{1/2} = 4^{1/2} (1 - x/4)^{1/2}$

$= 2 \left[1 - 1/2(x/4) - 1/8(x/4)^2 \right]$

$= 2 \left[1 - x/8 - x^2/128 \right]$

$= 2 - x/4 - x^2/64$

b) To make $\sqrt{4-x}$ approximate $\sqrt{3}$, $x = \frac{1}{4}$

$$\rightarrow 2 = \frac{1}{4} + \frac{1}{4}$$

$$= 1.734375 = 1.734 \text{ (3dp)}$$

5) a) $\sin(2x) = 2 \sin(x) \cos(x)$

b) $5 \sin(2x) + 3 \cos(2x) = 0$

$$5(2 \sin(x) \cos(x)) + 3 \cos(2x) = 0$$

$$\rightarrow 10 \sin(x) \cos(x) + 3 \cos(2x) = 0$$

$$\cos(x) [10 \sin(x) + 3] = 0$$

↓

$$\cos(x) = 0$$

$$\rightarrow x = 90^\circ, 270^\circ$$

↓

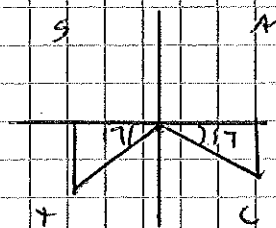
$$10 \sin(x) + 3 = 0$$

$$\rightarrow \sin(x) = -\frac{3}{10}$$

$$x = -17.45$$

$$x = 342.54, x = 197.45$$

$$= 342.5^\circ, 197.5^\circ \text{ (1dp)}$$



c) $\sin(2x) + \cos(2x) = 1 + \sin(x)$

$$2 \sin(x) \cos(x) + (1 - 2 \sin^2(x)) = 1 + \sin(x)$$

$$2 \sin(x) \cos(x) - 2 \sin^2(x) = \sin(x)$$

$$2 \sin(x) [\cos(x) - \sin(x)] = \sin(x)$$

$$2 [\cos(x) - \sin(x)] = 1$$

by $\sin(x)$

6) a) $x^2 y + y^3 = 2x + 1$

$$\rightarrow 2xy + x^2 y \frac{\partial y}{\partial x} + 3y^2 \frac{\partial y}{\partial x} = 2$$

when $x = 2, y = 1$

$$\rightarrow 4 + 4 \frac{\partial y}{\partial x} + 3 \frac{\partial y}{\partial x} = 2$$

$$7 \frac{\partial y}{\partial x} = -2$$

$$\frac{\partial y}{\partial x} = -\frac{2}{7}$$

$x^2 y$

$$u = x^2 \quad v = y$$

$$\frac{\partial u}{\partial x} = 2x \quad \frac{\partial v}{\partial x} = y \frac{\partial y}{\partial x}$$

$$\rightarrow 2xy + x^2 y \frac{\partial y}{\partial x}$$

b) At st points, $dy/dx = 0$

$\rightarrow 2xy = 2$ (from equation in a)

$\rightarrow y = 1/x$

Sub in to equation of curve:

$\rightarrow x^2 (1/x) + (1/x)^3 = 2x + 1$

$\rightarrow xc + 1/x^3 = 2x + 1$

$\rightarrow 1/x^3 = x + 1$

7) i) $dx/dt = -kt e^{1/2x}$

$\frac{1}{e^{1/2x}} dx = -kt dt$

$\int e^{-1/2x} dx = -k \int t dt$

$= -2e^{-1/2x} = -k \frac{t^2}{2} + c$

ii) $x=6, t=0 \rightarrow -2e^{-3} = 0 + c \rightarrow c = -2e^{-3}$

$-2e^{-1/2x} = -k \frac{t^2}{2} - 2e^{-3}$

$\therefore \text{by } -2$ ~~$\ln(-2e^{-1/2x}) = \ln(k \frac{t^2}{2} - 2e^{-3})$~~

$\rightarrow e^{-1/2x} = k \frac{t^2}{4} + e^{-3}$

$\ln(e^{-1/2x}) = \ln(k \frac{t^2}{4} + e^{-3})$

$\rightarrow -1/2x = \ln(k \frac{t^2}{4} + e^{-3})$

$\rightarrow x = -2 \ln(k \frac{t^2}{4} + e^{-3})$

b) i) $x = -2 \ln\left(\frac{0.004 \times 10^2}{4} + e^{-3}\right)$

$= 3.797... = 3797 = 3800$ (nearest 100)

ii) Need $x=0$

$\rightarrow 0 = -2 \ln\left(\frac{0.004 \times t^2}{4} + e^{-3}\right)$

$0 = \ln\left(\frac{0.004 \times t^2}{4} + e^{-3}\right)$

$\therefore \ln(1) = \frac{0.004 \times t^2}{4} + e^{-3}$

$$1 - e^{-3} = \frac{0.004 t^2}{4}$$

$$4(1 - e^{-3}) = 0.004 t^2$$

$$\rightarrow t^2 = \frac{4(1 - e^{-3})}{0.004}$$

$$= 30.8255... = 30.8 \text{ (1dp)}$$

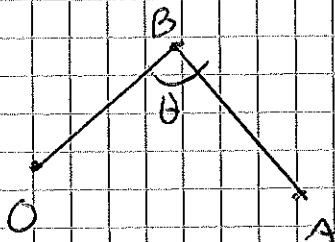
⑧ a) i) $\vec{AB} = \vec{AO} + \vec{OB}$

$$= \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

ii) ~~$|\vec{a}| = \sqrt{2^2 + 1^2 + (-1)^2} = \sqrt{6}$~~

$$|\vec{b}| = \sqrt{3^2 + 1^2 + (-2)^2} = \sqrt{14}$$

$$|\vec{AB}| = \sqrt{1^2 + 0^2 + (-1)^2} = \sqrt{2}$$



Need scalar product of $\vec{AB} \cdot \vec{OA}$

$$\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} = 3 + 0 + 2 = 5$$

$$a \cdot b = |a| |b| \cos(\theta)$$

$$\rightarrow 5 = \sqrt{14} \times \sqrt{2} \times \cos(\theta)$$

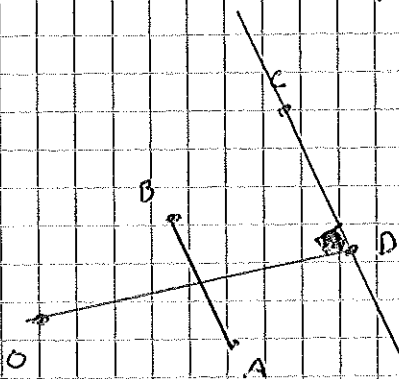
$$\rightarrow \cos(\theta) = \frac{5}{\sqrt{14} \sqrt{2}} = \frac{5}{\sqrt{28}} = \frac{5}{2\sqrt{7}}$$

b) $\vec{OC} = 2 \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ -4 \end{pmatrix}$

Parallel to \vec{AB}

$$\therefore \vec{r} = \begin{pmatrix} 6 \\ 2 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

c)



As D is on l ; $\vec{OD} = \begin{pmatrix} 6 + \lambda \\ 2 \\ -4 - \lambda \end{pmatrix}$

As DC is parallel to \vec{AB} and ODC is 90° :

$$\vec{OD} \cdot \vec{AB} = 0$$

$$\begin{pmatrix} 6 + \lambda \\ 2 \\ -4 - \lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = 6 + \lambda + 4 + \lambda = 0$$

$$\rightarrow 10 + 2\lambda = 0$$

$$\rightarrow \lambda = -5$$

$$\therefore \text{ODS} = \begin{pmatrix} 6 & -5 \\ 2 \\ -4 & -(-5) \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

\therefore co-ordinates of Δ are $(1, 2, 1)$